

# Strong Gravitational Lensing in a Charged Squashed Kaluza- Klein Black hole

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## Abstract

In this paper we investigate the strong gravitational lensing in a charged squashed Kaluza-Klein black hole. We suppose that the supermassive black hole in the galaxy center can be considered by a charged squashed Kaluza-Klein black hole and then we study the strong gravitational lensing theory and estimate the numerical values for parameters and observables of it. We explore the effects of the scale of extra dimension  $\rho_0$  and the charge of black hole  $\rho_q$  on these parameters and observables.

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## 1 Introduction

We know the light rays or photons would be deviated from their straight way when they pass close to the massive object such as black holes. This deflection of light rays well known as gravitational lensing. This gravitational lensing is one of the applications and results of general relativity [1] and is used as an instrument in astrophysics because it can help us to extract the information about stars. The basics of gravitational lensing theory developed by Liebes [2], Refsdal [3], Bourassa and Kantowski [4]. The gravitational lensing has been presented in details in [5] and reviewed by some papers (see for examples [6]-[9]). At this stage, the gravitational lensing is developed for weak field limit and could not describe some phenomena such as looping of light rays near the massive objects. Hence, scientists were

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starting to study these phenomena from another point of view and they posed gravitational lensing in strong field limit. Several studies about light rays close to the Schwarzschild horizon have been done in literatures: a semi-analytical investigation about geodesics in Kerr geometry has been made in [10], also the appearance of a black hole in front of a uniform background was studied in Refs. [11, 12]. Recently, Virbhadra and Ellis [13] shown when a source be highly aligned with Schwarzschild black hole and the observer, one set of infinitive relativistic images would produce on each side of black hole. These images are produced when the light ray passes with impact parameter near to its minimum and winds one or several times around the black hole before reaching to observer. In Refs [14, 15], the same problem was done with other methods. Afterwards, the same technique applied to other black holes [19]-[23], naked singularities and Janis-Newman-Winicour metric [23, 24].

Recently, the idea of large extra dimensions has attracted much attention [25] to construct theories in which gravity be unified with other forces. The five-dimensional Einstein-Maxwell theory with a Chern-Simons term [26] predicted five-dimensional charged black holes [27]. Such a higher-dimensional black holes would reside in a spacetime that is approximately isotropic in the vicinity of the black holes, but effectively four-dimensional far from the black holes [28]. We call higher-dimensional black holes with this property Kaluza-Klein black holes.

Presence of extra dimension is tested by quasinormal modes from the perturbation around the higher dimensional black hole [29] and the spectrum of Hawking radiation [30]. The gravitational lensing is another method to investigate the extra dimension. Thus, the study of strong gravitational lensing by higher dimensional black hole can help us to extract information about the extra dimension in astronomical observations in the future.

The Kaluza-Klein black holes with squashed horizon is one of the extra dimensional black holes and its Hawking radiation and quasinormal modes have been investigated in some papers [31]-[34]. Liu, Chen and Jing have studied the gravitational lensing by squashed Kaluza-Klein black holes in Refs [35, 36]. In Ref [35], the Author probed the effect of extra dimension on parameters and observables of strong gravitational lensing. Also variation of these parameters and observables with Gödel parameter and extra dimension is studied in a squashed Kaluza-Klein Gödel black hole in Ref [36]. In this paper we study the strong gravitational lensing in a charged squashed Kaluza-Klein black hole and peruse effects of the scale of extra dimension and charge of black hole on the coefficients and observables of strong gravitational lensing.

The rest of this paper is organized as follows: The Section 2 is briefly devoted to charged squashed Kaluza-Klein black hole metric. In Section 3 we use the Bozza's method [16, 17] to obtain the deflection angle and other parameters of strong gravitational lensing as well as variation of them with extra dimension and charge of black hole. In Section 4, we suppose that the supermassive object at the center of galaxy can be considered by the metric of charged squashed Kaluza-Klein black hole. Then we evaluate the numerical results for the coefficients and observables in the strong gravitational lensing. In the last Section, we present a summary of our work.

## 2 The charged squashed Kaluza- Klein black hole metric

The charged squashed Kaluza- Klein black hole spacetime is considered as the metric [37]

$$ds^2 = -f(r)dt^2 + \frac{k^2(r)}{f(r)}dr^2 + \frac{r^2}{4}[k(r)(\sigma_1^2 + \sigma_2^2) + \sigma_3^2], \quad (1)$$

where

$$\begin{aligned} \sigma_1 &= \cos \psi d\theta + \sin \psi \sin \theta d\phi, \\ \sigma_2 &= -\sin \psi d\theta + \cos \psi \sin \theta d\phi, \\ \sigma_3 &= d\psi + \cos \theta d\phi. \end{aligned} \quad (2)$$

The coordinates run in range,  $0 \leq \theta < \pi$ ,  $0 \leq \phi < 2\pi$ ,  $0 \leq \psi < 2\pi$  and  $0 < r < r_\infty$ . Moreover, the functions in the metric define as

$$f(r) = 1 - \frac{2M}{r^2} + \frac{q^2}{r^4}, \quad k(r) = \frac{f(r_\infty)r_\infty^4}{(r^2 - r_\infty^2)^2}. \quad (3)$$

Here  $M$  and  $q$  are the mass and charge of the black hole respectively . The killing horizon

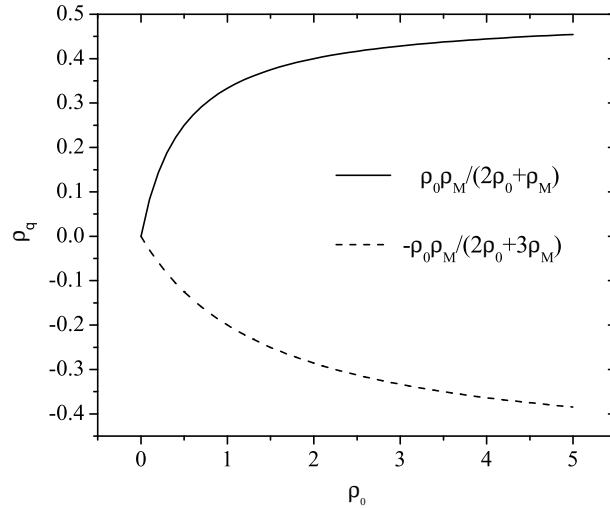


Figure 1: The figure shows the admissible values for  $\rho_0$  and  $\rho_q$ . Note that the region between two curves is allowed.

of the black hole is given by the equation  $f(r) = 0$ , that is  $r_h^2 = M \pm \sqrt{M^2 - q^2}$ . We see that the black hole have two horizons. As  $q \rightarrow 0$ , one has  $r_h^2 = 2M$ , which is the horizon of five-dimensional Schwarzschild black hole, and for  $M = q$  we have extremal black hole with a horizon  $r_{h\pm}^2 = M$ . Here we note that the argument of square root constraints the mass and charge values as,  $|M| \geq |q|$ . When  $r_\infty \rightarrow \infty$ , we have  $k(r) \rightarrow 1$ , which means that

the squashing effect disappears and the five-dimensional charged black hole is recovered. By using the transformations,  $\rho = \rho_0 \frac{r^2}{r_\infty^2 - r^2}$  and  $\tau = \sqrt{f(r_\infty)}t$ , metric (1) can be written in the following form

$$ds^2 = -\mathcal{F}(\rho)d\tau^2 + \frac{K(\rho)}{\mathcal{F}(\rho)}d\rho^2 + \mathcal{C}(\rho)(d\theta^2 + \sin^2\theta d\phi^2) + \mathcal{D}(\rho)\sigma_3^2, \quad (4)$$

$$\begin{aligned} \mathcal{F}(\rho) &= \left(1 - \frac{\rho_{h+}}{\rho}\right)\left(1 - \frac{\rho_{h-}}{\rho}\right), \\ K(\rho) &= 1 + \frac{\rho_0}{\rho}, \\ \mathcal{C}(\rho) &= \rho^2 K(\rho), \quad \mathcal{D}(\rho) = \frac{r_\infty^2}{4K(\rho)}, \end{aligned} \quad (5)$$

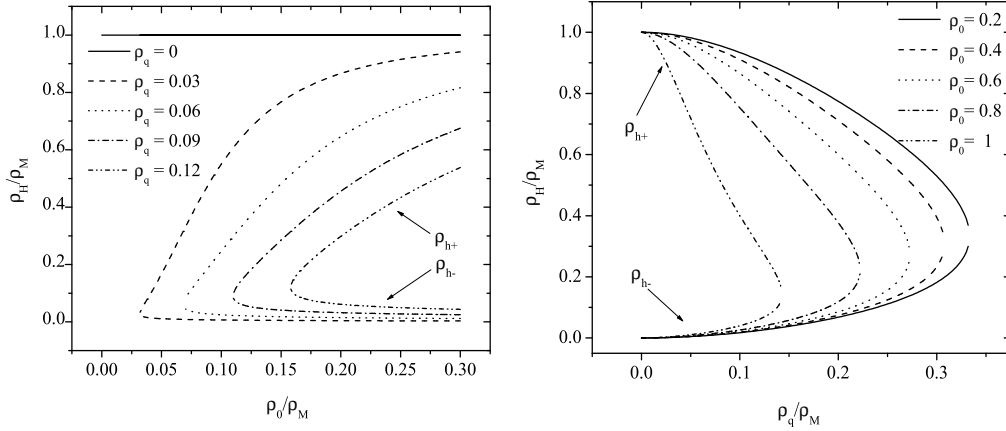


Figure 2: The plots show variation of radius of horizons with respect to  $\rho_0$  and  $\rho_q$ .

where  $\rho_{h+}$  and  $\rho_{h-}$  denote the outer and inner horizons of the black hole in the new coordinate and  $\rho_0$  is a scale of transition from five-dimensional spacetime to an effective four-dimensional one. By using coordinate transformation we can obtain  $\rho_{h\pm}$  in the following form

$$\rho_{h\pm} = \frac{2\rho_0}{2 - \eta \mp \xi} - \rho_0 \quad (6)$$

where

$$\eta = \frac{\rho_M}{\rho_0 + \rho_M}, \quad \xi = \sqrt{\frac{\rho_M^2}{(\rho_0 + \rho_M)^2} - \frac{4\rho_q^2}{(\rho_0 + \rho_q)^2}}, \quad (7)$$

with

$$\rho_M = \rho_0 \frac{2M}{r_\infty^2 - 2M}, \quad \rho_q = \rho_0 \frac{q}{r_\infty^2 - q}. \quad (8)$$

$\rho_q$	$\rho_0$	$\rho_0$	$\rho_q$
0	$\rho_0 > 0$	0	$\rho_q = 0$
0.03	$\rho_0 > 0.032$	0.2	$-0.059 < \rho_q < 0.143$
0.06	$\rho_0 > 0.068$	0.4	$-0.105 < \rho_q < 0.222$
0.09	$\rho_0 > 0.110$	0.6	$-0.143 < \rho_q < 0.273$
0.12	$\rho_0 > 0.158$	0.8	$-0.174 < \rho_q < 0.308$
0.15	$\rho_0 > 0.214$	1	$-0.200 < \rho_q < 0.333$

Table 1: Admissible values for  $\rho_0$  and  $\rho_q$  for constants values of  $\rho_q$  and  $\rho_0$  respectively.

Note that  $\rho_{h\pm} = \rho_0 \frac{r_{h\pm}^2}{r_\infty^2 - r_{h\pm}^2}$ . Here  $\rho_0^2 = \frac{r_\infty^2}{4} V(r_\infty)$ , so that  $r_\infty^2 = 4(\rho_0 + \rho_{h+})(\rho_0 + \rho_{h-})$ . The Komar mass of black hole is related to  $\rho_M$  with  $\rho_M = 2G_4 M$ , where  $G_4$  is the four dimensional gravitational constant. As  $\rho_q \rightarrow 0$ , the horizon of black hole becomes  $\rho_h = \rho_M$ , which is consistent with that in the neutral squashed Kaluza-Klein black hole [35]. We note that the square root in relation (7) constrains the values of  $\rho_0$  and  $\rho_q$ . For  $\rho_q = -\frac{\rho_0 \rho_M}{2\rho_0 + 3\rho_M}$  and  $\rho_q = \frac{\rho_0 \rho_M}{2\rho_0 + \rho_M}$ , the extremal black hole is obtained and admissible values for  $\rho_q$  are between these two values. We have plotted the admissible region for  $\rho_0$  and  $\rho_q$  in figure 1. Also we have evaluated the admissible values for specific values,  $\rho_0$  and  $\rho_q$ , in table 1. Hereafter, our figures are plotted by considering the table 1. Variation of the radius of horizons are plotted in figure 2.

### 3 Deflection angle in the strong field limit

In this section, we will investigate deflection angle of the light rays when they pass close to a charged squashed Kaluza-Klein black hole and probe the effect of the charge parameter  $\rho_q$  and the scale of extra dimension  $\rho_0$  on the deflection angle and coefficients of it in the equatorial plan  $\theta = \pi/2$ .

We consider the metric for charged squashed Kaluza-Klein black hole as

$$ds^2 = -\mathcal{F}(\rho)d\tau^2 + \mathcal{B}(\rho)d\rho^2 + \mathcal{C}(\rho)d\phi^2 + \mathcal{D}(\rho)d\psi^2, \quad (9)$$

where

$$\mathcal{B}(\rho) = \frac{K(\rho)}{\mathcal{F}(\rho)}. \quad (10)$$

The null geodesics equations are

$$\frac{dv_i}{dk} + \Gamma_{jk}^i v^j v^k = 0, \quad (11)$$

where

$$g_{ij}v^i v^j = 0. \quad (12)$$

$v^i = \frac{dx^i}{dk}$  is the tangent vector to the null geodesics and  $k$  is affine parameter. The following equations can be obtained from (11),

$$\begin{aligned}\frac{dt}{dk} &= \frac{E}{\mathcal{F}(\rho)}, \\ \frac{d\phi}{dk} &= \frac{L_\phi}{\mathcal{C}(\rho)}, \\ \frac{d\psi}{dk} &= \frac{L_\psi}{\mathcal{D}(\rho)},\end{aligned}\tag{13}$$

$$\left(\frac{d\rho}{dk}\right)^2 = \frac{1}{\mathcal{B}(\rho)} \left[ \frac{\mathcal{D}(\rho)E - \mathcal{F}(\rho)L_\psi^2}{\mathcal{F}(\rho)\mathcal{D}(\rho)} - \frac{L_\phi^2}{\mathcal{C}(\rho)} \right].\tag{14}$$

Here  $E$ ,  $L_\phi$  and  $L_\psi$  are constants of motion. Also, the  $\theta$ -component of equation (11) in the equatorial plan,  $\theta = \pi/2$ , leads to

$$\frac{d\phi}{dk}(\mathcal{D}(\rho)\frac{d\psi}{dk}) = 0.\tag{15}$$

So this constraint implies that either  $\frac{d\phi}{dk} = 0$  or  $L_\psi = \mathcal{D}(\rho)\frac{d\psi}{dk} = 0$ . Now, we set  $L_\psi = 0$ , as done in Refs. [36, 35].

From equation (14), the impact parameter and photon sphere equation can be obtained as

$$u = \sqrt{\frac{\mathcal{C}(\rho)}{\mathcal{F}(\rho)}},\tag{16}$$

and

$$\mathcal{F}(\rho)\mathcal{C}'(\rho) - \mathcal{C}(\rho)\mathcal{F}'(\rho) = 0.\tag{17}$$

The above equation is very complicated to solve analytically and thus we have calculated it numerically. Variations of radius of photon sphere are plotted with respect to the charge  $\rho_q$  and the scale of extra dimension  $\rho_0$  in the figure 3. Figure 4 shows variation of impact parameter in it's minimum value (at radius of photon sphere) with respect to the same parameters. These figures show that by adding the charge to the black hole the behavior of radius of photon sphere and minimum of impact parameter is different in compare with the neutral black hole [35]. As  $\rho_0$  approaches to it's minimum values in the table 1, the radius of photon sphere and impact parameter become divergent and when  $\rho_0$  increases,  $\rho_{sp}$  and  $u_{sp}$  increase up to a maximum and descend. Also, when  $\rho_q$  increases then,  $\rho_{sp}$  and  $u_{sp}$  increase for smaller  $\rho_0$  and decrease for the larger  $\rho_0$ . Variation of the impact parameter and radius of photon sphere are similar, but note that for  $\rho_q = 0$ , when  $\rho_0 \rightarrow \infty$ , the photon sphere curve tends to  $\rho_{sp} = 2$ , while the impact parameter goes to infinity.

The deflection angle in the charged squashed Kaluza-Klein black hole can be written as

$$\alpha(\rho_s) = I(\rho_s) - \pi,\tag{18}$$

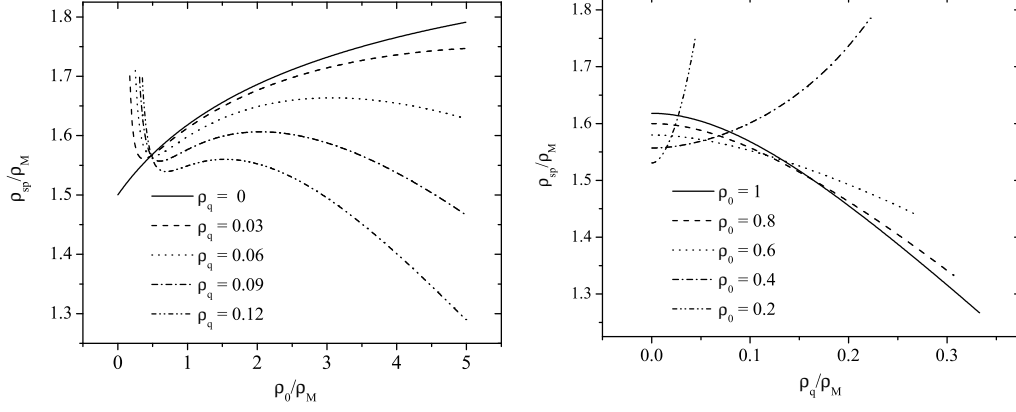


Figure 3: The figures show the variation of radius of photon sphere with respect to  $\rho_0$  and  $\rho_q$ .

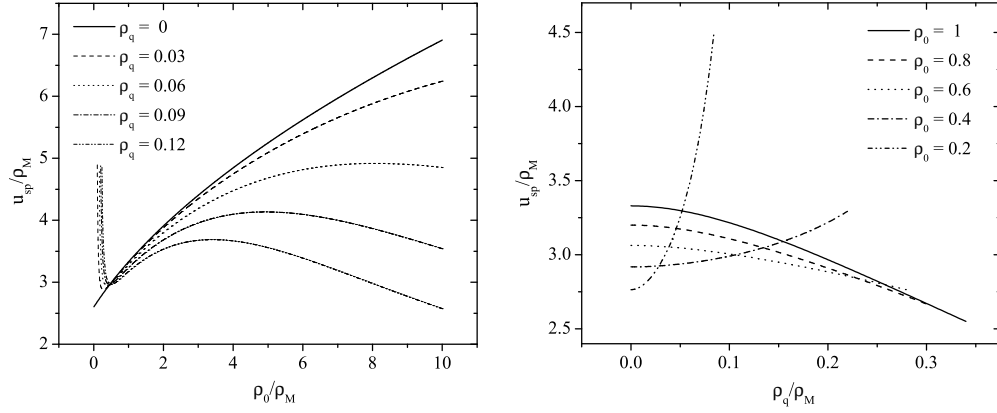


Figure 4: Variations of impact parameter in the charged squashed Kaluza-Klein black hole.

where  $\rho_s$  is the closest approach and

$$I(\alpha) = 2 \int_{\rho_s}^{\infty} \left[ \frac{\mathcal{C}(\rho)}{\mathcal{C}(\rho_s)} \mathcal{F}(\rho_s) - \mathcal{F}(\rho) \right]^{-\frac{1}{2}} \frac{d\rho}{\rho}. \quad (19)$$

When we decrease the  $\rho_s$  (and consequently  $u$ ) the deflection angle increases. At some points, the deflection angle will exceed from  $2\pi$  so that the light ray makes a complete loop around the compact object before reaching the observer. By decreasing  $\rho_s$  further, the photon will wind several times before emerging. Finally, for  $\rho_s = \rho_{sp}$  the deflection angle diverges and the photon is captured by the black hole.

We can rewrite the equation (19) as

$$I(\rho_s) = 2 \int_0^1 R(z, \rho_s) f(z, \rho_s) dz, \quad (20)$$

with

$$R(z, \rho_s) = 2 \frac{\rho}{\rho_s} \sqrt{\frac{\mathcal{C}(\rho_s)}{\mathcal{C}(\rho)}}, \quad (21)$$

and

$$f(z, \rho_s) = \frac{1}{\sqrt{\mathcal{F}(\rho_s) - \mathcal{F} \frac{\mathcal{C}(\rho_s)}{\mathcal{C}(\rho)}}}, \quad (22)$$

where we have defined  $z = 1 - \frac{\rho_s}{\rho}$ . The function  $R(z, \rho_s)$  is regular for all values of  $z$  and  $\rho_s$ , while  $f(z, \rho_s)$  diverges as  $z$  approaches to zero. Therefore, we can split the integral (19) in two part, the divergent part  $I_D(\rho_s)$  and the regular one  $I_R(\rho_s)$ , as follows

$$I_D(\rho_s) = \int_0^1 R(0, \rho_{sp}) f_0(z, \rho_s) dz, \quad (23)$$

$$I_R(\rho_s) = \int_0^1 [R(z, \rho_s) f(z, \rho_s) - R(0, \rho_{sp}) f_0(z, \rho_s)] dz. \quad (24)$$

Here we expand the argument of the square root in  $f(z, \rho_s)$  up to the second order in  $z$  [36]

$$f_0(z, \rho_s) = \frac{1}{\sqrt{p(\rho_s)z + q(\rho_s)z^2}}, \quad (25)$$

where

$$p(\rho_s) = \frac{\rho_s}{\mathcal{C}(\rho_s)} [\mathcal{C}'(\rho_s) \mathcal{F}(\rho_s) - \mathcal{C}(\rho_s) \mathcal{F}'(\rho_s)], \quad (26)$$

$$q(\rho_s) = \frac{\rho_s^2}{2\mathcal{C}(\rho_s)} [2\mathcal{C}'(\rho_s) \mathcal{C}(\rho_s) \mathcal{F}'(\rho_s) - 2\mathcal{C}'(\rho_s)^2 \mathcal{F}(\rho_s) + \mathcal{F}(\rho_s) \mathcal{C}(\rho_s) \mathcal{C}''(\rho_s) - \mathcal{C}^2(\rho_s) \mathcal{F}''(\rho_s)]. \quad (27)$$

For  $\rho_s > \rho_{sp}$ ,  $p(\rho_s)$  is nonzero and the leading order of the divergence in  $f_0$  is  $z^{-1/2}$ , which have a finite result. As  $\rho_s \rightarrow \rho_{sp}$ ,  $p(\rho_s)$  approaches zero and divergence is of order  $z^{-1}$ ,



that makes the integral divergent. Therefor, the deflection angle can be approximated in the following form [23]

$$\alpha(\theta) = -\bar{a} \log \left( \frac{\theta D_{OL}}{u_{sp}} - 1 \right) + \bar{b} + O(u - u_{sp}), \quad (28)$$

where

$$\begin{aligned} \bar{a} &= \frac{R(0, \rho_{sp})}{2\sqrt{q(\rho_{sp})}}, \\ \bar{b} &= -\pi + b_R + \bar{a} \log \frac{\rho_{sp}^2 [\mathcal{C}''(\rho_{sp})\mathcal{F}(\rho_{sp}) - \mathcal{C}(\rho_{sp})\mathcal{F}''(\rho_{sp})]}{u_{sp}\sqrt{\mathcal{F}^3(\rho_{sp})\mathcal{C}(\rho_{sp})}}, \\ b_R &= I_R(\rho_{sp}), \quad u_{sp} = \sqrt{\frac{\mathcal{C}(\rho_{sp})}{\mathcal{F}(\rho_{sp})}}. \end{aligned} \quad (29)$$

The parameter  $D_{OL}$  is the distance between an observer and gravitational lens. Using of (28) and (29), we can investigate the properties of strong gravitational lensing in the charged squashed Kaluza- Klein black hole. In this case, variations of the  $u_{sp}$ , the coefficients  $\bar{a}$  and  $\bar{b}$ , and the deflection angle  $\alpha(\theta)$  have been plotted with respect to the extra dimension  $\rho_0$ , and charge of the black hole  $\rho_q$ , in figures 5-7.

In figures 5 and 6, we see that for fixed  $\rho_q$ , the coefficient  $\bar{a}$  increases with the size of the extra dimension and the coefficient  $\bar{b}$  increases for the smaller  $\rho_q$  and decreases for the larger ones. Moreover,  $\bar{a}$  decreases with increasing  $\rho_q$  for fixed  $\rho_0$ . The coefficient  $\bar{b}$  increases up to a maximum and descends for smaller  $\rho_0$  but increases monotonically for larger  $\rho_0$ . Variation of deflection angle is presented in figure 7. One can see that the deflection angle increases with extra dimension and decreases with  $\rho_q$ . We note that, as  $\rho_0$  approaches to it's minimum values in left column of table 1, the coefficients  $\bar{a}$ ,  $\bar{b}$  and deflection angle  $\alpha(\theta)$  tend to zero. By comparing these parameters with those in four-dimensional schwarzschild and Reissner-Nordström black holes , we could extract information about the size of extra dimension as well as the charge of black hole by using strong field gravitational lensing.

## 4 Observables in the strong field limit

Now, we are going to study the effects of the scale parameter  $\rho_0$  and charge of black hole  $\rho_q$  on the observables in the strong gravitational lensing. If we suppose that the spacetime of the supersessive black hole at the galaxy center of Milky Way can be considered by a charged squashed Kaluza-Klein black hole then, we can estimate the numerical values for the coefficients and observables of gravitational lensing in the strong field limit.

We can write the lens equation in strong gravitational lensing, as the source, lens, and observer are highly aligned as follows [15]

$$\beta = \theta - \frac{D_{LS}}{D_{OS}} \Delta\alpha_n, \quad (30)$$

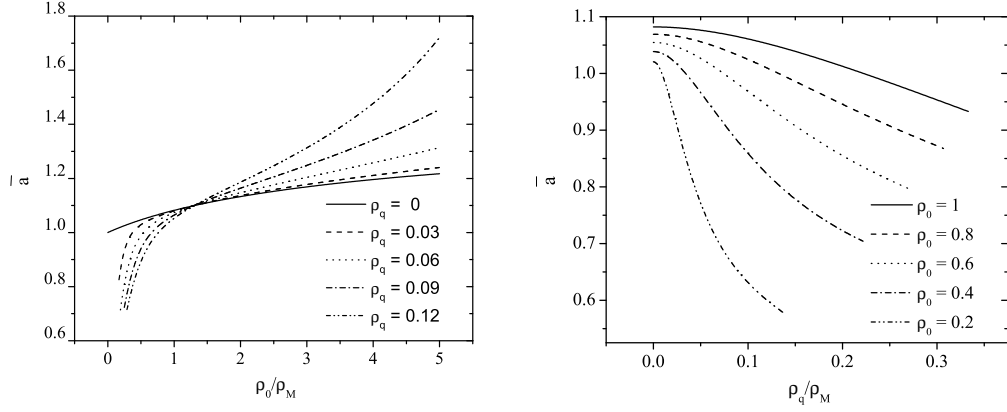


Figure 5: Variation of the coefficient  $\bar{a}$  with respect to the scale parameter  $\rho_0$  and charge of black hole  $\rho_q$ .

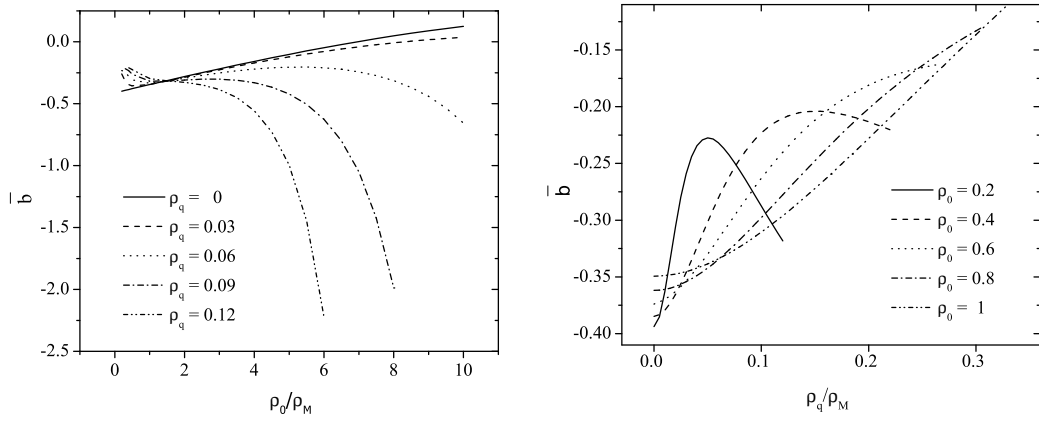


Figure 6: Variation of the coefficient  $\bar{b}$  with respect to the scale parameter  $\rho_0$  and charge of black hole  $\rho_q$ .

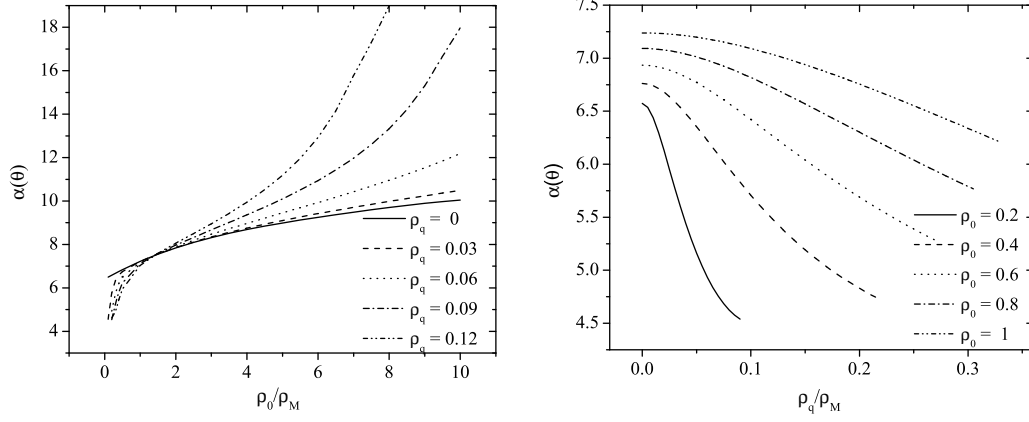


Figure 7: The plots show the variation of deflection angle with  $\rho_0$  and  $\rho_q$  in the charged squashed Kaluza-Klein black hole. ( we considered  $u = u_{sp} + 0.003$  )

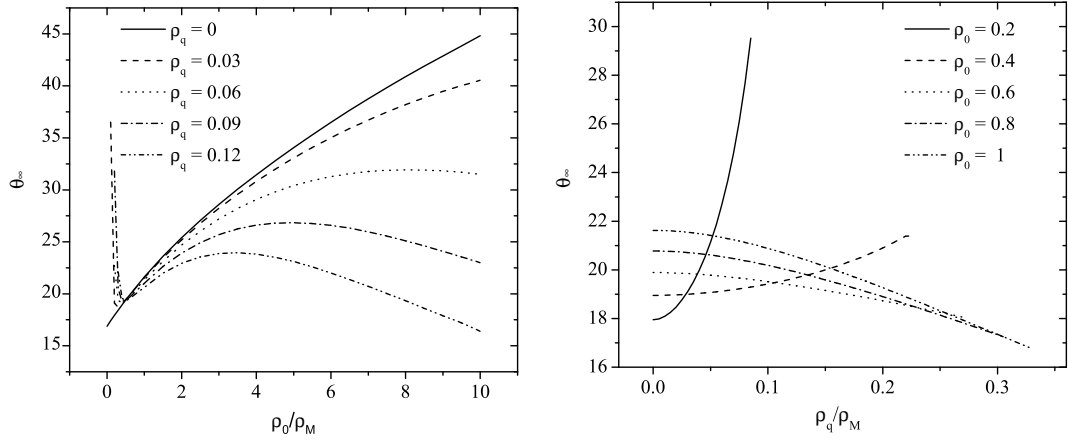


Figure 8: Variation of the angular position  $\theta_\infty$  with the scale parameter  $\rho_0$  and charge of black hole  $\rho_q$ .

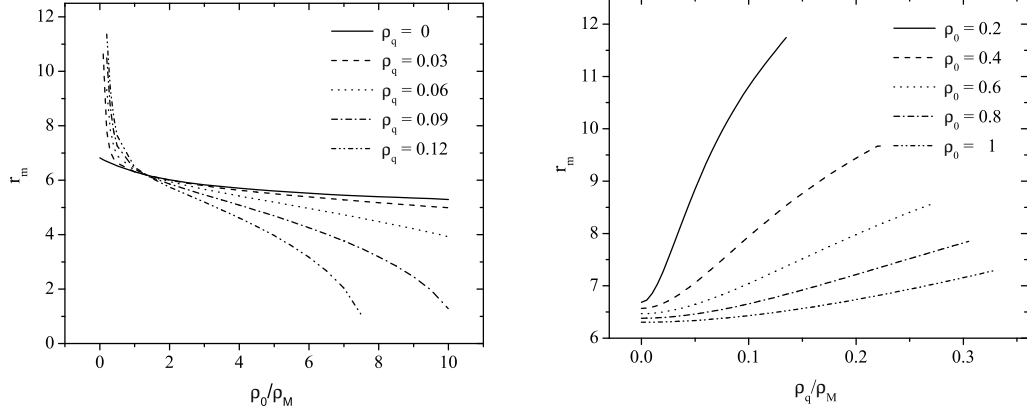


Figure 9: Variation of the relative magnitudes  $r_m$  with the scale parameter  $\rho_0$  and charge of black hole  $\rho_q$ .

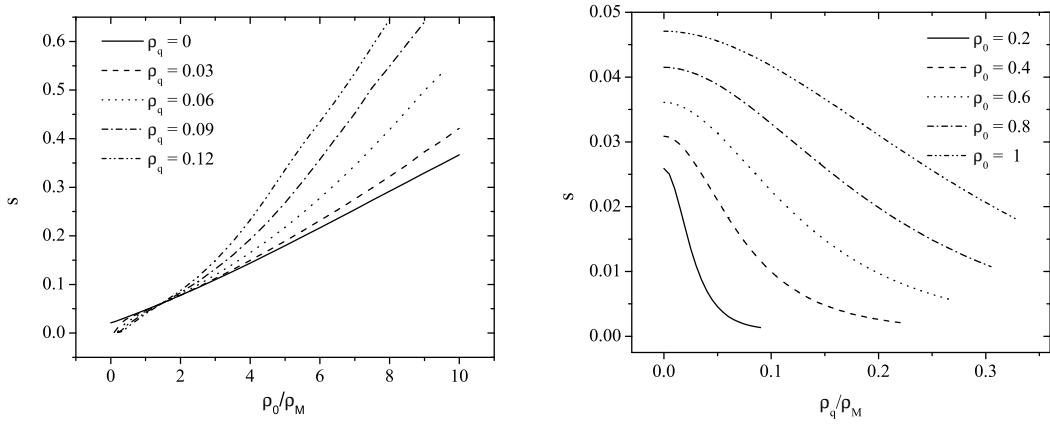


Figure 10: Variation of the angular separation  $s$  with the scale parameter  $\rho_0$  and charge of black hole  $\rho_q$ .

where  $D_{LS}$  is the distance between the lens and source.  $D_{OS}$  is the distance between the observer and the source so that,  $D_{OS} = D_{LS} + D_{OL}$ .  $\beta$  and  $\theta$  are the angular position of the source and the image with respect to lens, respectively.  $\Delta\alpha_n = \alpha - 2n\pi$  is the offset of deflection angle with integer  $n$  which indicate the  $n$ -th image.

The  $n$ -th image position  $\theta_n$  and the  $n$ -th image magnification  $\mu_n$  can be approximated as follows [23, 15]

$$\theta_n = \theta_n^0 + \frac{u_{sp}(\beta - \theta_n^0)e^{\frac{\bar{b}-2n\pi}{\bar{a}}}D_{OS}}{\bar{a}D_{LS}D_{OL}}, \quad (31)$$

$$\mu_n = \frac{u_{sp}^2(1 + e^{\frac{\bar{b}-2n\pi}{\bar{a}}})e^{\frac{\bar{b}-2n\pi}{\bar{a}}}D_{OS}}{\bar{a}\beta D_{LS}D_{OL}^2}. \quad (32)$$

$\theta_n^0$  is the angular position of  $\alpha = 2n\pi$ . In the limit  $n \rightarrow \infty$ , the relation between the minimum of impact parameter  $u_{sp}$  and asymptotic position of a set of images  $\theta_\infty$  can be expressed by  $u_{sp} = D_{OL}\theta_\infty$ . In order to obtain the coefficients  $\bar{a}$  and  $\bar{b}$ , in the simplest case, we separate the outermost image  $\theta_1$  and all the remaining ones which packed together at  $\theta_\infty$ , as done in Refs [23, 15]. Thus  $s = \theta_1 - \theta_\infty$  is considered as the angular separation between the first image and other ones and the ratio of the flux of them is given by

$$\mathcal{R} = \frac{\mu_1}{\sum_{n=2}^{\infty} \mu_n}. \quad (33)$$

We can simplify the observables and rewrite them in the following form

$$s = \theta_\infty e^{\frac{\bar{b}}{\bar{a}} - \frac{2\pi}{\bar{a}}}, \quad \mathcal{R} = e^{\frac{2\pi}{\bar{a}}}. \quad (34)$$

Thus, by measuring the  $s$ ,  $\mathcal{R}$  and  $\theta_\infty$ , one can obtain the values of the coefficients  $\bar{a}$ ,  $\bar{b}$  and  $u_{sp}$ . If we compare these values by those obtained in the previous section, we could detect the size of the extra dimension and charge of black hole. Another observable for gravitational lensing is relative magnification of the outermost relativistic image with the other ones. This observable is showed by  $r_m$  which is related to  $\mathcal{R}$  by

$$r_m = 2.5 \log \mathcal{R}. \quad (35)$$

Using  $\theta_\infty = \frac{u_{sp}}{D_{OL}}$  and equations (29), (34) and (35) we can estimate the values of the coefficients  $\bar{a}$  and  $\bar{b}$ , in the strong field gravitational lensing. Variation of the observables  $\theta_\infty$ ,  $s$  and  $r_m$  are plotted in figures 8-10. Note that the mass of the central object of our galaxy is estimated to be  $2.8 \times 10^6 M_\odot$  and the distance between the sun and the center of galaxy is  $D_{OL} = 8.5 \text{ kpc}$  [13].

For the different  $\rho_0$  and  $\rho_q$ , the numerical values for main observables and the strong field limit coefficients for the black hole at center of our galaxy which is supposed to be described by the charged squashed Kaluza-Klein black hole are listed in Table 2. One can see that our results reduce to those in the four-dimensional Schwarzschild black hole as  $\rho_0 = 0$  and also our results are in agreement with the results of Ref. [35] in the limit  $\rho_q \rightarrow 0$ .

## 5 Summary

The extra dimension is one of the important predictions in string theory which is believed to be a promising candidate for the unified theory. The five-dimensional Einstein-Maxwell theory with a Chern-Simons term in string theory predicted five-dimensional charged black holes. We considered the charged squashed Kaluza-Klein black hole spacetime and investigated the strong gravitational lensing by this metric. We obtained theoretically the deflection angle and other parameters of strong gravitational lensing and studied variation of them with respect to extra dimension and charge of black hole. Finally we estimated numerically the values of observables i.e. relativistic images  $\theta_\infty$ , the angular separation  $s$  and the relative magnitudes  $r_m$  for different  $\rho_0$  and  $\rho_q$ . Our results are presented in figures 1-10 and Table 2. These results may help us to detect the extra dimension and charge of black hole by astronomical observations in the future.

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$\rho_q$	$\rho_0$	$\theta_\infty$ ( $\mu$ arcsec)	$s$ ( $\mu$ arcsec)	$r_m$ (magnitudes)	$\bar{a}$	$b$
Schwarzschild $\longrightarrow$	0	16.869	0.02111	6.8218	1.0	-0.4003
0	0.2	17.949	0.02588	6.6838	1.0206	-0.3938
	0.4	18.950	0.03088	6.5678	1.0387	-0.3847
	0.6	19.890	0.03610	6.4681	1.0547	-0.3738
	0.8	20.778	0.04150	6.3813	1.0690	-0.3619
	1	21.623	0.04708	6.3046	1.0820	-0.3493
0.03	0.2	19.098	0.01091	7.787	0.8760	-0.2590
	0.4	18.998	0.02638	6.7706	1.0076	-0.3460
	0.6	19.848	0.03416	6.5376	1.0435	-0.3585
	0.8	20.714	0.04047	6.4113	1.0640	-0.3543
	1	21.546	0.04651	6.3174	1.0799	-0.3452
0.06	0.2	22.698	0.00307	9.3261	0.7315	-0.2318
	0.4	19.127	0.01828	7.2268	0.9440	-0.2805
	0.6	19.737	0.02691	6.7145	1.0160	-0.3231
	0.8	20.541	0.03780	6.4921	1.0508	-0.3346
	1	21.333	0.04495	6.3531	1.0738	-0.3340
0.09	0.2	31.798	0.00133	10.4912	0.6502	-0.2718
	0.4	19.332	0.01164	7.7627	0.8788	-0.2333
	0.6	19.576	0.02422	6.9563	0.9807	-0.2825
	0.8	20.284	0.03415	6.6107	1.0319	-0.3076
	1	21.011	0.04263	6.4082	1.0646	-0.3172
0.12	0.2	72.561	0.00121	11.3647	0.6003	-0.3184
	0.4	19.619	0.00736	8.2875	0.8232	-0.2101
	0.6	19.379	0.01915	7.2306	0.9435	-0.2455
	0.8	19.662	0.03008	6.7562	1.0097	-0.2776
	1	20.606	0.03980	6.4801	1.0527	-0.2961

Table 2: Numerical estimations for the coefficients and observables of strong gravitational lensing by considering the supermmasive object of galactic center be a charged squashed Kaluza-Klein black hole.